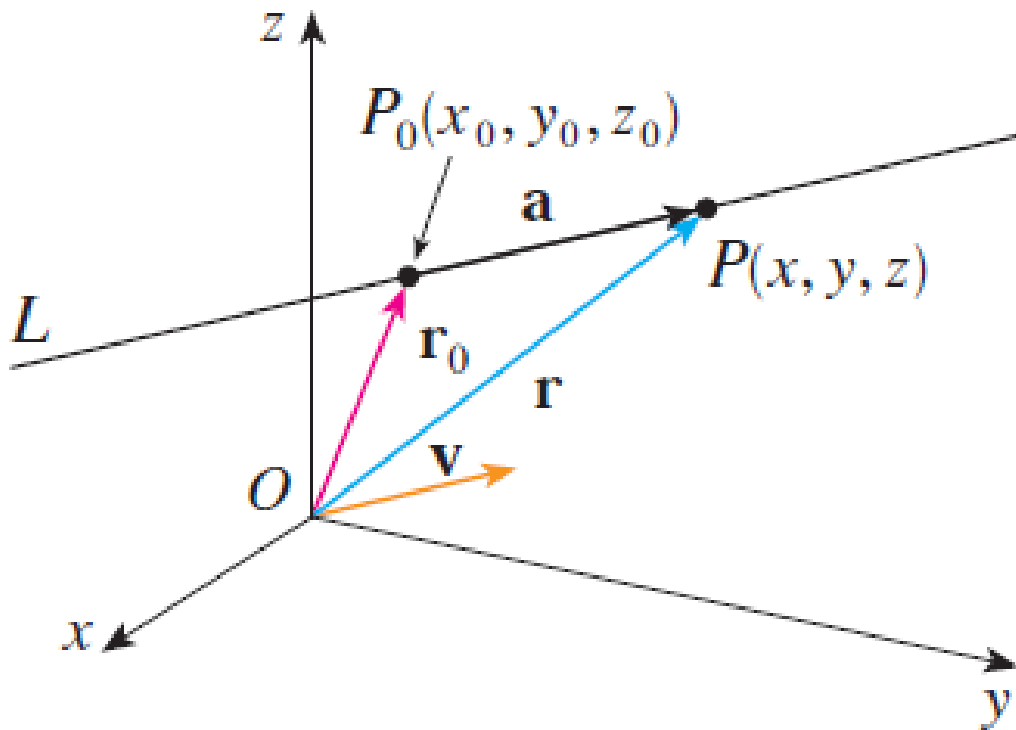


Visual of a Line in 3D



Notes: In the picture,

\mathbf{v} = a vector parallel to the line

$\mathbf{r}_0 = \langle x_0, y_0, z_0 \rangle$

= a vector that points from the origin to some particular point (x_0, y_0, z_0) on the line.

Recall, any scale multiple of \mathbf{v} will be parallel to \mathbf{v} . So consider a vector \mathbf{a} that can be written as $\mathbf{a} = t\mathbf{v}$ (that is the vector \mathbf{a} in the picture).

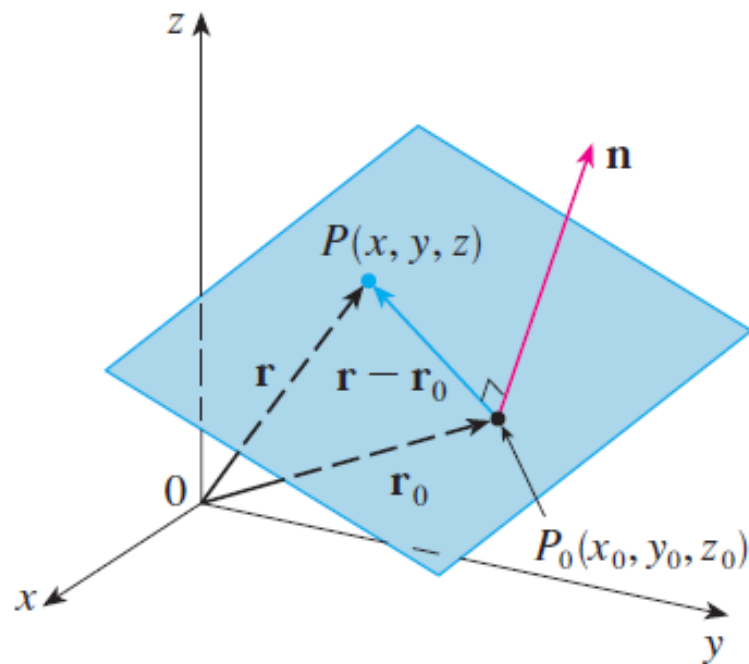
Since \mathbf{a} is parallel to \mathbf{v} which is parallel to the line, if we add \mathbf{a} to \mathbf{r}_0 , then it will give another point on the line.

That is, if $\mathbf{r}_0 + \mathbf{a} = \langle x, y, z \rangle = \mathbf{r}$, then (x, y, z) is also on the line.

ALL points on the line can be obtained by doing the same thing with different values of t . Thus, all points (x, y, z) on the line satisfy

$\mathbf{r} = \langle x, y, z \rangle = \mathbf{r}_0 + t\mathbf{v}$ (**the vector form of the 3D line equation**)
for some scale multiple t .

Visual of a Plane in 3D



Notes: In the picture,

\mathbf{n} = a vector perpendicular to the plane (a *normal* vector)

$\mathbf{r}_0 = \langle x_0, y_0, z_0 \rangle$

= a vector that points from the origin to some particular point (x_0, y_0, z_0) on the plane.

Let (x, y, z) be any other point on the plane.

Consider the vector that points from (x_0, y_0, z_0) to (x, y, z) , which is

$\langle x - x_0, y - y_0, z - z_0 \rangle$ (which is denoted by $\mathbf{r} - \mathbf{r}_0$ in the picture)

Key Observation: Since \mathbf{n} is perpendicular to the plane, that means that it must be perpendicular to $\mathbf{r} - \mathbf{r}_0$.

Thus,

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0 \quad \text{(the vector form of the 3D plane equation)}$$